American University of Beirut
Statistics 230 Sections 3 and 4
Final Exam, January 30^{th}, 2010

NAME: SECTION:

PART I. Answer Questions 1 to 8 by circling T for TRUE or F for FALSE. (1 point each)

1. **T** **F** There exists a pair of random variables X and Y such that $E[X] = E[Y] = 0$, $E[XY] = 3$, $var(X) = 1$ and $var(Y) = 3$.

2. **T** **F** The equation $std.dev(X_1 + \cdots + X_n) = std.dev(X_1) + \cdots + std.dev(X_n)$ is true if and only if $X_1, \cdots, X_n$ are independent.

3. **T** **F** We can reconstruct the joint p.d.f of two random variables X and Y by knowing the two marginal densities of X and Y.

4. **T** **F** Let X and Y be independent Poisson variables with distinct parameters λ_1 and λ_2. Then the sum $X + Y$ also has a Poisson distribution.

5. **T** **F** Let X and Y be independent exponential random variables with the same parameter λ. Then the variable $Z = max(X, Y)$ is also exponentially distributed.

6. **T** **F** Let X and Y be two independent random variables uniformly distributed in $[0, 1]$. Let $M_1 = min(X, Y)$ and $M_2 = max(X, Y)$. Then M_1 and M_2 are independent.

7. **T** **F** Given that the moment generating function of random variable X is $M(t) = \frac{1}{9}(1 + 2t)^2$, then $E[X]$ is less than 1.

8. **T** **F** Consider strings that only contain H (for heads) or T (for tails) resulting from tossing a coin repeatedly. The number s_n of strings of size $n > 2$ that contain the substring HHT exactly once at the very end is governed by the recurrence relation $s_n = s_{n-1} + s_{n-2} + 1$.

PART II. SHORT ANSWER QUESTIONS: Answer the following question by giving a brief justification. You can use the other blank side of the page to finish your work if needed. (2points each)

9. Let X be the number of days that a patient needs to stay in a certain hospital. Suppose X has the p.m.f. $f(x) = \dfrac{6-x}{15}$, $x = 1, \cdots, 5$. The patient receives \$100 from his insurance company for *each* of the first three days in the hospital and \$50 for any other day, if needed. Find the expected payment by the insurance company.

$ 220

10. Find $P[X > Y]$ where X and Y are two random variables with the joint p.d.f.

$$f(x) = \begin{cases} 12xy & \text{for } 0 \le x \le 1, x^2 \le y \le \sqrt{x} \\ 0 & \text{otherwise} \end{cases}.$$

1/2

11. Given $E[X+3] = 9$ and $E[(X+3)^2] = 100$. Find the standard deviation of X

sqrt
(19)

12. An urn contains 3 white chips and 7 black chips. 8 chips are selected *with replacement* from the urn. Let X be the number of white chips observed among the selected chips. The variance of X is

1.68,
x~b(8,0.3)

13. The probability of suffering a side effect from a certain medication is 0.002. If 500 persons are given this medication, the approximate probability that more than one person suffers a side effect is

0.26
poisson approx.

14. Let X be a random variable that is normally distributed with $\mu = 0$ and $\sigma = 5$. Find a point a so that $P[-a < X < a] = 0.8$.

???????

-> normal, mu =0 ==> centered
at 0 : p=2*p(x<a)
==> p(0<x<a)=0.4

numerically, a=6.40

15. The cumulative distribution function of a random variable X is given by $F(x) = 1-e^{-2x}$. Find the median of X.

0.346574

16. X is a random variable with p.d.f. $f(x) = 3x^2$, $0 < x < 1$. The distribution of $Y = \sqrt{X}$ is

6*y^5

DOMAINE : 0<y<1

17. Let X and Y have the joint density $f(x, y) = 2$, for $0 < y \leq x < 1$. Find the conditional density of Y given $X = 1/2$.

2

PART III. PUT the detailed solutions to TWO of the following problems. Indicate clearly the problem that you omit. (3 points each)

18. Let X and Y be two random variables with the joint p.d.f.

$$f(x) = \begin{cases} 2x & \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases}.$$

(a) Find $\rho(X, Y)$, the correlation coefficient of X and Y.

(b) Find the regression line that approximates linearly the relationship between X and Y.

(a):0

(b): Y = mu_y = 0.5

19. Let X_1 and X_2 be two independent and exponentially distributed random variables with common parameter $\theta = 1$. Consider the transformation $Y_1 = X_1/X_2$ and $Y_2 = X_1$.

(a) Find the joint density of Y_1 and Y_2.

(b) Deduce the probability density function of the ratio X_1/X_2

(y(2))/(y(1)^1)   * e^(-y2(1+1/y1))

1/(y1^2+1+2y1)

20. Let $X_1, \cdots, X_{36}$ be independent and exponentially distributed random variables with parameter $\theta = 5$.

(a) Approximate the probability that the average $\bar{X}$ exceeds 6.

(b) Use the moment generating function technique to find the distribution of $\sum_{k=1}^{36} X_k$.